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The Quantile Zeid-*G* Family of Distributions by a Lambert-*W* Type Weight with Illustration to Bladder Cancer Patients Data

Clement Boateng Ampadu[1*]

*[*1]Carrolton Road, Boston, MA 02132-6303, USA.*

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ABSTRACT

Some contributions to quantile distribution theory appeared in [1-2]. In this short note, the class of quantile Zeid-G statistical distributions are introduced. A sub-model of this family is shown to be practically significant in fitting real-life data. The researchers are asked to further develop the properties and applications of this new class.

Keywords: $(\frac{1}{e})^{\alpha}$ PT-G family distributions, Zeid-*G*, quantile generated family of distributions, cancer patient's data, heavy-tailed Ampadu-*G*

INTRODUCTION

Firstly, we recall the following

Definition 1.1. [3]. Let $\alpha \neq (\frac{1}{e})$,$\alpha > (\frac{1}{e})$ and $\xi > 0$ where α is the rate scale parameter and ξ is a vector of parameters in the baseline distribution all of whose entries are positive. A random variable Z is said to follow the $(\frac{1}{e})^{\alpha}$ power transform family of distributions if the Cumulative Distribution Function (CDF) is given by

$$F\,(x;\ \alpha,\xi) = \frac{1-\ e^{-\alpha G\,(x;\,\xi)}}{1-e^{-\alpha}} \quad x \in R,$$

where the baseline distribution has *CDF G (x; ξ)*.

Proposition 1.2 [3]. *The PDF of the* $(\frac{1}{e})^{\alpha}$ *power transform family distribution is given by*

$$f\,(x;\alpha,\xi) = \frac{1}{1-e^{-\alpha}}\,(\alpha g(x,\xi))e^{-\alpha g(x,\xi)}$$

where $\alpha \neq \frac{1}{e}$ *,* $\alpha > \frac{1}{e}$ *and* $\xi > 0$ *and G(x;ξ) is the baseline cumulative distribution with probability density function g(x; ξ).*

Remark 1.3. *The parameter space for α have been relaxed to α ∈ R, α ≠ 0. The relaxation has been employed in several papers, and for example, see [4]*

Using this relaxation, we introduced the following, inspired by the structure of the Chen-G CDF [5].

Definition 1.4 *[6]. A random variable Y is called a Zeid random variable, if the CDF is given by Zeid-G, that is,*

$$F(x;\alpha,\beta,\xi) = \frac{1-e^{-\alpha\left\{1-e^{-\beta G(x;\xi)}\right\}}}{1-e^{-\alpha\left\{1-e^{-\beta}\right\}}} \qquad x \in Supp(G),$$

where the baseline distribution has CDF G (x;ξ), ξ is a vector of parameters in the baseline CDF whose support depends on the chosen baseline CDF. α, β ∈ R, and α, β ≠ 0

The Zeid-Normal distribution was shown to be a good fit to Table 2 [7]. On the other hand, in [8], we introduced the following:

Theorem 1.5 [8]. *Let X follow Ampadu- {Standard Uniform}, and put*

$$Y = Q_G(e^{1-X})$$

where $Q_G = G^{-1}$ *(.) is the quantile function of the distribution with baseline CDF G, then the CDF of Y is*

$$F_Y(y;\alpha,\xi) = 1 - \frac{1-e^{-\alpha(1-\log\,(G(y;\xi)))^2}}{1-e^{-\alpha}}$$

Moreover, for any t>0, we have

$$\lim_{y\to\infty} e^{ty}\left(1-F_Y(y;\alpha,\xi)\right) = \infty$$

Corresponding author: Clement Boateng Ampadu, 31 Carrolton Road, Boston, MA 02132-6303, USA, Tel: +1-617-469-7268; E-mail: drampadu@hotmail.com

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Thus, Y is a heavy-tailed Ampadu-G random variable.

In particular, we showed the Heavy-tailed Ampadu-Weibull distribution was a good fit to the bladder cancer patients data recorded in [9]. Motivated by these developments, this paper introduces a so-called quantile Zeid-G family of distributions. Moreover, in the special case G follows the Heavy-tailed Ampadu-Weibull distribution, we show the quantile Zeid-{Heavy-tailed Ampadu-Weibull} distribution is a good fit to the bladder cancer patients data recorded [9]. The reader is recommended to explore further properties and applications of the new class of distributions presented in this paper.

The New Family Defined

Recall that the generic form of the quantile CDF in the sense of [1] and [2] is Q [V (F (x))] where Q is a quantile, V is an appropriate weight, and F (x) is some baseline distribution.

Remark 2.1. *If in Definition 1.4, α = β = 1, and G is the CDF of the uniform distribution on [0, 1], then we say Y is a standard Zeid random variable.*

Our "Q" is a special solution obtained by solving the following equation for y

$$x = \frac{1 - e^{e^{-y}-1}}{1 - e^{e^{\frac{1}{e}}-1}}$$

In particular, the special solution, our "Q", is given by

$$Q(x) = \log\left(\frac{1}{1 - \log\left(\frac{e}{e^{\frac{1}{e}}x - ex + e}\right)}\right)$$

Our "V" is obtained by solving the following equation for y

$$y * e^{1-y} = \frac{1 - e^{e^{-x-1}}}{1 - e^{e^{\frac{1}{e}}-1}}$$

In particular, our "V" is given by,

$$V(x) = -W\left(\frac{e - e^{e^{-x}}}{e\left(e^{\frac{1}{e}} - e\right)}\right)$$

Where *W(z)* gives the principal solution for m in $z = me^m$. Thus, we introduce the following

Definition 2.2. *The CDF of the quantile Zeid-G family of distributions is given by*

$$K(x;\xi) = \log\left(-\frac{1}{\log\left(\frac{1}{\left(e-e^{\frac{1}{e}}\right)W\left(\frac{e-e^{e^{-G(x;\xi)}}}{e\left(e^{\frac{1}{e}}-e\right)}\right)+e}\right)}\right)$$

Where G is some baseline distribution, x ∈ Supp (G), ξ is a vector of parameters in the baseline distribution whose support depends on G, and W (z) gives the principal solution for m in z =me^m.

PRACTICAL ILLUSTRATION

Assume the baseline distribution is given by the heavy-tailed Ampadu-Weibuldistribution, that is, in Definition 2.2 we set

$$G(x;\xi) = 1 - \frac{1 - e^{-\alpha e^{-2\left(\frac{x}{b}\right)^{a}}}}{1 - e^{-\alpha}}$$

Where x, a, b > 0, 0≠ $\alpha \in \mathbb{R}$

Remark 3.1. *We write QZHT AW (a, b, α) for short to represent the Quantile Zeid- {heavy-tailed Ampadu Weibull} distribution*

The quantile Zeid- {heavy-tailed Ampadu Weibull} distribution appears a good fit to the bladder cancer patient's data recorded in [9] as shown in **Figure 1**.

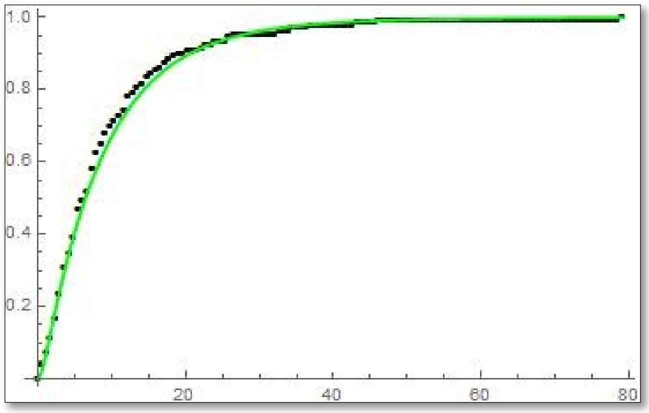

Figure 1. The CDF of QZHT AW (0.616938, 2.25327, 6.18023) fitted to the empirical distribution of the bladder cancer patient's data recordedin [9].

CONCLUSION

In this paper, we introduced the quantile Zeid-G family of distributions, and showed the quantile Zeid- {Heavy-tailed Ampadu-Weibull} distribution is a good fit to real life data. We hope the researchers will further develop the properties and applications of this new class of statistical distributions.

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